PG – 215

IV Semester M.Sc. Degree Examination, June 2017
(CBCS)
MATHEMATICS
M 403 T(A) : Magnetohydrodynamics

Time : 3 Hours Max. Marks : 70

Instructions : 1) *Answer* **any five** *questions.*
2) **All** *questions carry* **equal** *marks.*

1. a) Write a short note on :
   i) Electrostatic and electromagnetic units
   ii) Rationalized mks system.

   b) Derive the conservation of free charges in the form $\frac{\partial \rho_e}{\partial t} + \frac{\sigma}{\varepsilon}\rho_e = 0$ and hence find the relaxation time. (6+8)

2. In the case of an applied magnetic field derive the boundary conditions on its normal and tangential components at the interface between two media. 14

3. a) Derive the magnetic induction equation in the form $\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{q} \times \vec{B}) + \frac{1}{R_m}\nabla^2 \vec{B}$ where the quantities have their usual meaning. Discuss the nature of this equation under the limiting cases of (i) $R_m << 1$ and (ii) $R_m >> 1$.

   b) Write a short note on magnetic Reynolds number. (11+3)

4. a) Explain the concept of frozen-in phenomenon. Establish this fact by proving the appropriate theorem.

   b) Derive the analogue of Helmholtz vorticity equation of MHD. (8+6)

5. For a force-free magnetic field $\vec{B}$, prove the following

   i) $\int_V B^2 dv = 0$, if $\vec{B} = 0$ on the surface S enclosing the volume V.

   ii) $\alpha = \frac{\nabla\phi.(\nabla\chi \times \nabla\eta)}{|\nabla\phi|^2 - \chi|\nabla\eta|^2}$, where $\vec{B} = \nabla\phi + \chi\nabla\eta$.

   iii) $\nabla^2\vec{B} + \alpha^2\vec{B} = \vec{B} \times \nabla\alpha$, where α is an abnormality parameter. (8+3+3)

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6. a) Explain the Bennet pinch and discuss kink and sausage mode of instabilities associated with it.

   b) Show that the Lorentz force can be expressed as a sum of two surface forces. Explain how one of these forces is the cause for the propagation of Alfven waves in a conducting fluid. (8+6)

7. a) Discuss important applications of Alfven waves pertaining to geophysical and astrophysical contexts.

   b) Establish one-dimensional classical Alfven wave equations in their standard form. (6+8)

8. a) Write a short note on (i) Hartmann flow (ii) Hartmann number and (ii) Prandtl number.

   b) Obtain the velocity distribution for the hydromagnetic plane Couette flow and hence deduce the velocity distribution for plane Couette flow. Sketch the velocity profiles and discuss the effect of magnetic field on the same. (6+8)
